SECOND EDITION.

—THE—

XXth Century ARITHMETIC

ARITHMETICAL COMPUTATIONS AND BUSINESS CALCULATIONS BY MODERN METHODS.

PHILOSOPHICAL IN ARRANGEMENT, NATURAL IN PROGRESSIVENESS, AND PRACTICAL IN APPLICATION.

— BY —

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INVOLUTION.

ORAL EXERCISES.

1. What is the product of 6 used twice as a factor?
2. What is the product of 4 used 4 times as a factor?
3. What is the product of $\frac{3}{4}$ used 4 times as a factor?
4. What will be obtained by using .05 twice as factor?
5. A hall is 65 ft. square; how many sq. ft. in floor?
6. A block of granite measures 6 feet on each edge; how many cubic feet does it contain?

DEFINITIONS.

A POWER is the product obtained by multiplying a number by itself, or using it as a factor. Thus, 4, (2×2), is the second power of 2; 8, $(2\times 2\times 2)$, is the third power of 2.

INVOLUTION is the process of finding any power of a number.

The EXPONENT of a power is a small figure placed at the right and a little above the factor to show how many times it is to be used. Thus, 2^2 shows that 2 is to be raised to the second power; 3^3 shows that 3 is to be raised to the third power, etc.

The SQUARE of a number is the second power of the number.

The CUBE of a number is the third power of the number.

1. Find the third power or cube of 15; also of 1.5.

$$15^3 = 15 \times 15 \times 15 = 3375.$$
$$1.5^3 = 1.5 \times 1.5 \times 1.5 = 3.375.$$

2. Find the cube of $\frac{2}{3}$; of $\frac{3}{4}$; of $\frac{1}{2}$; of $\frac{4}{5}$; of $\frac{2}{5}$.

$$(\tfrac{2}{3})^3 = \tfrac{2}{3} \times \tfrac{2}{3} \times \tfrac{2}{3} = \tfrac{8}{27}.$$

3. What is the value of 5^3? of 7^4? of 8^2? of 9^3?
4. Fine the cube of 6^2; the square of 6^3.
5. What is the product of 4^3 by 6^2? of 5^3 by 8^3?

RULE.—*Separate the numbers into periods of three figures each, beginning at the unit's place.*

The first figure of the root will be the cube root of the greatest cube contained in the left-hand period. Subtract this cube from the left-hand period, and to the remainder annex the next period to form a dividend.

For a trial divisor take three times the square of the root figure regarded as tens, and divide the dividend by it to obtain the second figure of the root.

For a complete divisor add to the trial divisor three times the second figure of the root multiplied by the first figure considered as tens, and the square of the second figure of the root. From the dividend subtract the product of the complete divisor by the second figure of the root.

Bring down next period, if any, and proceed as before to find trial divisor, treating the part of root already found as a single figure expressing tens.

NOTE.—If the trial divisor is not contained in the dividend, annex a cipher in the root, two ciphers to the trial divisor, and another period to the dividend. Then divide the dividend by the trial divisor for the next figure of the root.

Find the cube root of the following numbers:

1.	6359.	4.	32768.	7.	.373248	10.	$\frac{512}{4096}$.
2.	39304.	5.	205.379	8.	.091125	11.	$\frac{2197}{2744}$.
3.	15625.	6.	12.812904	9.	.216	12.	$\frac{343}{2197}$.

13. Find the cube root of 2 to three decimal places.
14. A cubical block of granite contains 21952 cubic feet; what is the length of its side?
15. How many square feet in one face of a cube containing $91\frac{1}{8}$ cubic feet? 336 sq. ft.
16. Find the cube root of 5 to three decimal places.
17. $\sqrt[3]{103.823} = ?$ $\sqrt[3]{54.872} = ?$

3×20^2 is alone almost equal to the first factor. Dividing 9576 by 3×20^2, or 1200, gives 7 as the depth of these facing solids.

But since 3×20^2 is somewhat smaller than the first of the two factors, 7 is probably larger than the second factor. Taking 6 as more likely to be the other factor, the volume of the solids composing Fig. 2 may be calculated as follows: $[(3 \times 20^2)+3 \times (20 \times 6)+6^2] \times 6=9576$. This being equal to the remainder of the cube whose volume is 17576, 6 is the addition to be made to the edge of the cube whose volume is 12000 to give the edge of the cube whose volume is 17576. Adding 6 to 20 gives 26, the edge required.

The extraction of the cube root may be abridged by omitting unnecessary figures. To show this, let it be

2. Required to find the cube root of 390457.604629.

PROCESS.

	390'457.604'629 (73.09
	343
14700	47 457
630	
9	
15339	46 017
1598700	1 440 604
159870000	1 440 604 629
197100	
81	
160067181	1 440 604 629

EXPLANATION.—Dividing first remainder, 47457, by 3 times the square of 7 regarded as tens, that is, by 14700, gives 3 as the second figure of the root. Adding to 14700, the trial divisor, 3 times 70×3, or 630, and 3^2, gives 15339, the complete divisor. This multiplied by the second root figure, and the product subtracted from 47457, gives 1440, to which the next period is annexed.

Taking 3 times the square of 73, regarded as tens, for the next trial divisor, we find it is not contained in 1440604. A cipher is, therefore, written in the root, and the last period brought down.

Taking 3 times the square of 730, regarded as tens, as the next trial divisor, or what is the same, annexing two ciphers to the previous trial divisor, and dividing by it as before, gives 9 as the final root figure. Completing the trial divisor by adding to it 3 times 730×9, and 9^2, and multiplying the divisor so completed by 9, gives a product equal to the remainder of the number whose cube is to be found. Hence, 73 09 is the cube root required.

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EVOLUTION.

ORAL EXERCISES.

1. What are two equal factors of 4? Of 9? Of 16?
2. What are three equal factors of 8? Of 27? Of 64?
3. 25 is the second power of what number? 36? 49?

DEFINITIONS.

A ROOT of a number is one of two or more equal factors that will, if multiplied together, produce the number.

The SQUARE ROOT of a number is one of *two* equal factors whose product equals the number; the CUBE ROOT, one of *three* equal factors, etc.

EVOLUTION is the process of finding any required root of a number. It is the *reverse* of *Involution*.

The RADICAL SIGN, $\surd$, is placed before and over a number to indicate that the root is to be extracted.

The INDEX of the root is a small figure placed over the radical sign to indicate the root required. Thus, $\sqrt[3]{29}$ indicates the cube root of 29. When no root is written over the radical, the square root is understood.

NOTE.—The root required may be indicated by a fractional exponent. Thus, $36^{\frac{1}{2}}$ means $\sqrt{36}$, and $64^{\frac{2}{3}}$ means $\sqrt[3]{64^2}$. The numerator of the fraction indicates the power and the denominator shows what root of that power is to be found.

A PERFECT POWER is a number whose root can be exactly extracted. Thus, 25 is a perfect power whose square root is 5.

An IMPERFECT POWER is a number whose root can not be exactly extracted. Thus 10, whose square root is 3.162+.

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SQUARE ROOT.

The square of a number contains twice as many figures, or one less than twice as many, as the number itself. Thus,

Numbers:	1,	7,	20,	7.6,	.55,	200.
Squares:	1,	49,	400,	57.76,	.3025,	40000.

Hence, *If the square of a number be separated into periods of two figures each, beginning at the right, there will be as many periods as there are figures in the square root of the number.*

Note 1.—The left-hand period may contain only one figure.

Note 2.—Decimals are separated into periods by beginning at the decimal point. If the right-hand period is not full, a cipher is added.
Thus, 2,46.35,82; 14.25,30; 1,36.74,20.

1. Required to show the composition of the square of 26.

PROCESS.

Explanation.—26 may be written $20+6$. Hence, $26^2 = (20+6)^2$. Writing the factors as in the margin, $20+6$ is first multiplied by 20, and then by 6, and the partial products are added.

$$\begin{array}{l} 20 + 6 \\ 20 + 6 \\ \hline 20^2 + (20 \times 6) \\ \qquad (20 \times 6) + 6^2 \\ \hline 20^2 + 2 \times (20 \times 6) + 6^2 = \end{array}$$

$$20^2 + [(2 \times 20) + 6] \times 6 = 676.$$

Hence, *The square of any number composed of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

Note.—Twice the product of the tens by the units, plus the square of the units, is the same as the sum of twice the tens and the units, multiplied by the units.

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subtracting 8000 from 17576, leaves 9576. This remainder consists of two factors, the first of which is 3 times the square of the tens, plus 3 times the product of the tens by the units, plus the square of the units. The second factor is the units' figure as yet unknown. In the first factor, 3 times the square of the tens is alone almost equal to the entire factor. Taking 3 times the square of the tens, or 1200, as one of two factors whose product equals the remainder, 9576, the other factor is equal to 9576÷1200, or 7. But since 1200 is too small, 7 is likely to be too large. Taking 6 as more probably the correct units' figure, and completing the trial divisor by adding to it 3 times the product of the tens by the units, and the square of the units, gives 1596 as the complete divisor. Multiplying 1596 by 6 gives a product equal to the remainder 9576, showing that 17576 is a perfect cube, whose cube root is 26.

GEOMETRICAL ILLUSTRATION.

Fig. 1.

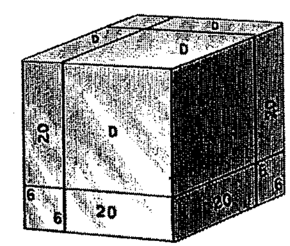
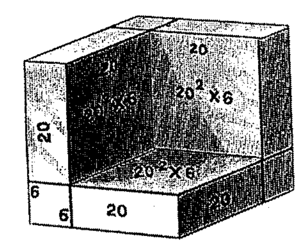


Fig. 2.



Let Fig. 1 be a cube whose volume is 17576. Since the volume of a cube is the product of three equal factors, the cube root of the volume will be one of the factors, that is, the edge of the cube. If the edge of the cube were 20, its volume would be 8000, and if it were 30, the volume would be 27000. The volume being between 8000 and 27000, the edge must be between 20 and 30. Removing a cube whose edge is 20, the remainder of Fig. 1, shown by Fig. 2, has a volume equal to 17576 − 8000, or 9576. This remainder is made up of three rectangular solids each 20 by 20 on the face, of three others each 20 long with a width and depth equal to the excess of the edge of Fig. 1 above 20, and a cube whose edge is equal to the same excess. The depth of all these being the same, their united volumes, or the remainder, 9576, is equal to $(3\times20^2+3\times20\times depth + depth^2)\times depth$. Neither of these two factors is entirely known, but

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NOTE 1. – The left-hand period may contain one, two, or three figures.

NOTE 2.—Decimals are separated into periods by beginning at the decimal point. The right-hand period must contain three figures, ciphers being annexed if necessary.

TO ANALYZE THE CUBE OF A NUMBER.

1. Required to show the composition of the cube of 26.

PROCESS.

$$26^2 = 20^2 + 2\times(20\times 6) + 6^2$$
$$20 \ + 6$$
$$20^3 + 2\times(20^2\times 6) + \quad (20\times 6^2)$$
$$(20^2\times 6) + 2\times(20\times 6^2) + 6^3$$
$$20^3 + 3\times(20^2\times 6) + 3\times(20\times 6^2) + 6^3 =$$
$$20^3 + [(3\times 20^2) + (3\times 20\times 6) + 6^2]\times 6.$$

EXPLANATION.—It has been shown that $26^2 = 20^2 + 2\times(20\times 6) + 6^2$. Multiplying this by $20+6$ will give the cube of $20+6$.

HENCE, *The cube of any number composed of tens and units is equal to the cube of the tens, plus three times the square of the tens multiplied by the units, plus three times the tens multiplied by the square of the units, plus the cube of the units.*

TO FIND THE CUBE ROOT OF A NUMBER.

1. Find the cube root of 17576.

PROCESS.

	17,576	(20+6
	8 000	
$3\times 20^2 = 1200$	9 576	
$3\times 20\times 6 = 360$		
$6^2 = 36$		
1596	9 576	

EXPLANATION.—Separating 17576 into periods of three figures each, beginning at the right, gives two periods. Hence, there are two figures in the cube root. Whatever the tens' figure of the root is, the significant figures of its cube are included in the left-hand period. The largest cube in the left-hand period is 8000, and its cube root is 20. Writing 20 as the tens of the root, and

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TO FIND THE SQUARE ROOT OF A NUMBER.

1. Find the square root of 676.

SOLUTION.

	6,76	(20+6=26
	4 00	
40+6=46)	2 76	
	2 76	

EXPLANATION. — Separating the factor into periods, we find there will be two figures in the square root. The tens of the root must be found from the first period, 600. The largest square in this is 400 and its square root is 20. Writing 20 as the tens of the root, and subtracting 400 from 676, leaves 276. This remainder consists of twice the tens plus the units, multiplied by the units. One of these factors, twice the tens, or 40, is known; the other, which is the unit's figure of the root, is found by dividing 276 by 40, giving 6 as the unit's figure sought. Multiplying twice the tens, or 40, by 6, the unit's figure, and the unit's figure by itself, that is, multiplying 46 by 6, gives 276. This being equal to the remainder of the square, shows that 676 is a perfect square, of which 26 is the square root.

GEOMETRICAL ILLUSTRATION.

Fig. 1.

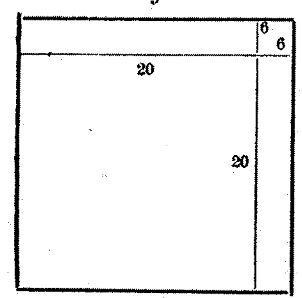
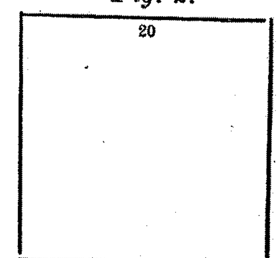


Fig. 2.



Let Fig. 1 be a square whose area is 676. Since the area of a square is the product of two equal factors, the square root of th area will be the side of the square. If the side were 20, the ar would be 400; and if 30, the area would be 900. The area bei between 400 and 900 the side must be between 20 and 30. Removi

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from Fig. 1 a square 20 on a side (Fig. 2), the remainder of Fig. 1 must have an area of 676−400, equal to 276. This remainder consists of a small square, and two rectangles whose united length is 20 × 2, or 40, and whose width added to the side of the square removed will give the side of Fig. 1. These two rectangles form the greater part of the remaining area, 276. Dividing 276 by 40, the united length of the two rectangles, gives 6 as the probable width of the rectangles. Since 276 is somewhat greater than the united areas of the two rectangles, 6 may be greater than their width. Two rectangles, each 20 long and 6 wide, and a square 6 on a side, are together equal to $2 \times (20 \times 6) + 6^2$, or 276. This being equal to the remainder of the square, 6 is the width of the rectangles. Adding 6 to the side of the square whose area is 400, gives 26 as the side of the square whose area is 676.

In practice, unnecessary figures are omitted.

2. Find the square root of 196.8409.

SOLUTION.

```
         1,96.84,09 (14.03
         1
  24)     96
          96
  2803)   8409
          8409
```

EXPLANATION.—After bringing down the third period, and forming the trial divisor, 28, the next figure of the root is 0 because 28 is not contained in 8. Therefore, 0 is placed in both the root and the divisor, and 09, the next period, brought down.

RULE.—*Separate the given number into periods of two figures each, beginning at unit's place.*

Find the greatest square in the left-hand period, and place its root in the quotient. Subtract the square from the left-hand period, and to the remainder annex the next period.

Divide this dividend, omitting the figure on the right, by double the part of the root already found, annex quotient to root, also to trial divisor; then multiply the divisor thus completed by the figure of the root last

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obtained, and subtract the product from the dividend. Bring down next period, if any, and proceed as before.

If the trial divisor is not contained in the remainder, place a cipher in the root, also in trial divisor, and bring down another period.

The decimal places in the root should be as many as the decimal periods.

The square root of a fraction is found by extracting the square root of its numerator and of its denominator, and writing them as the numerator and denominator of the root.

Find the square root of the following numbers:

1. 576.	5. 11881.	9. 5387041.	13. .042025
2. 4096.	6. 14884.	10. 4260.854	14. 1162.25
3. 9604.	7. 186624.	11. 196.1369	15. 86.9432
4. 7569.	8. 994009.	12. .59049	16. 462.25

17. $\sqrt{\frac{64}{81}}$; $\left(\frac{225}{324}\right)^{\frac{1}{2}}$; $\sqrt{\frac{900}{1225}}$; $\left(\frac{1600}{2025}\right)^{\frac{1}{2}}$; $\sqrt{\frac{90601}{130321.}}$

18. How many rods in length is each side of a square field, which contains 40 acres?

19. A field is 77 rods long and 32 rods wide. What is the length of a side of a square containing an equal area? 48 rods.

CUBE ROOT.

The cube of a number contains three times as many figures as the number itself, three times as many figures less one, or three times as many figures less two. Thus,

Numbers:	1	7	20	7.6	.55
Cubes:	1	343	8000	440.176	.166375

HENCE, *If the cube of any number be separated into periods of three figures each, beginning at the right, there will be as many periods as there are figures in the cube root.*

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